Angular Power Spectrum Estimation of Cosmic Ray Anisotropies with Full or Partial Sky Coverage

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Abstract

We study the angular power spectrum estimate in order to search for large scale anisotropies in the arrival directions distribution of the highest-energy cosmic rays. We show that this estimate can be performed even in the case of partial sky coverage and validated over the full sky under the assumption that the observed fluctuations are statistically spatial stationary. If this hypothesis - which can be tested directly on the data - is not satisfied, it would prove, of course, that the cosmic ray sky is non isotropic but also that the power spectrum is not an appropriate tool to represent its anisotropies, whatever the sky coverage available. We apply the method to simulations of the Pierre Auger Observatory, reconstructing an input power spectrum with the Southern site only and with both Northern and Southern ones. Finally, we show the improvement that a full-sky observatory brings to test an isotropic distribution, and we discuss the sensitivity of the Pierre Auger Observatory to large scale anisotropies.

1 Introduction

The origin of the highest energy cosmic rays is a theoretical challenge of modern astrophysics, and is subject of much experimental efforts. Above 10^{20} eV, the current data are too scarce for one to make any definitive statement about the existence or the lack of the GZK cutoff, as well as a statistically meaningful information about the arrival direction distribution. Whereas the AGASA

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experiment (Takeda et al. 2003) is over since January 2004, a new generation of experiments especially dedicated to the highest energies is emerging, and the first of them is the Pierre Auger Observatory currently under construction (The Pierre Auger collaboration 1995). For a recent review on the state of the art of the highest-energy cosmic rays, we refer the reader to (Cronin 2004) for instance.

The distribution of the arrival directions is certainly one of the most crucial observable in order to yield some evidences about the sources of the highest-energy cosmic rays (Isola et al. 2002; Sigl et al. 2003; Sigl et al. 2004). When trying to point out large scale anisotropies, one is naturally led to work with the angular power spectrum of the arrival direction distribution. The detection of large scale anisotropy could probe certain classes of sources and/or test certain propagation models in presence of magnetic fields to be associated with such large scale celestial patterns. In addition, the evidence for large scale anisotropy around 1 EeV[1] claimed by the AGASA collaboration (Hayashida et al. 1998; Hayashida et al. 1999) motivates even more this kind of studies.

From the PeV energy range, the flux of cosmic rays is so low that we need ground based experiments with large collecting areas measuring the secondary products of the interaction of the cosmic ray in the upper atmosphere. As any ground based experiment has only at one's disposal a limited field of view in declination distribution, anisotropy analysis are generally done owing to the nearly uniform exposure in right ascension by using a 1-dimensional coordinate system instead of the natural 2-dimensional one over the sphere. This is the case for example of the Rayleigh formalism (Linsley 1975) which necessarily corrupts the sensitivity to tiny anisotropies. In order to exploit the angular power spectrum analysis methods, it is assumed within the cosmic rays community that a full exposure of the sky is required (Sommers 2000; Anchordoqui et al. 2003). The aim of this paper is to show that this conclusion arises only because of the choice of the spherical harmonic coefficients estimate, and to show that with another choice of estimate, standard anisotropy analysis methods can be used even with a partial and non-uniform coverage of the celestial sphere.

By denoting $\vec{n}_i$ each cosmic ray arrival direction, the standard estimate of the spherical harmonic coefficients is computed through

$$a_{\ell m} = \frac{1}{\mathcal{A}} \sum_{i=1}^{N} \frac{Y_{\ell m}(\vec{n}_i)}{\omega(\vec{n}_i)}$$

where N is the total number of events, $\omega(\vec{n})$ is the relative exposure func-

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[1] 1 EeV$\equiv 10^{18}$ eV

tion of the considered experiment, and $\mathcal{A}$ a normalization constant taken as $\sum_{i=1}^{N} 1/\omega(\vec{n}_i)$. As well known, the use of $1/\omega$ allows for decoupling the modes when working with a variable exposure over the whole celestial sphere, but breaks down in case of partial exposure of the sky, because it is no longer possible to perform the full sky integrations that are required to measure the multi-poles of the celestial cosmic ray intensity (Sommers 2000).

In this paper, we choose to introduce and adapt the quadratic estimator method that is widely used in the Cosmic Microwave Background analysis (see e.g. (Hivon et al. 2002)) where effects of a partial exposure can be deconvoluted from the observations in order to recover the true underlying power spectrum. We show that the application of this method allows for the standard anisotropy analysis with an exposure possibly going to zero in some parts of the sky. This point is of major interest for most cosmic rays experiments, as the Southern site of the Auger Observatory for instance.

Another approach to power spectrum estimation is through maximum likelihood (see (Bond et al. 1998; Borrill et al. 1999) and (Hamilton 2003) for a review) which has the advantage of solving exactly the problem. It however requires an explicit representation of the sky covariance and is computationally very time consuming and numerically hard to achieve on large datasets. The quadratic estimate proposed here avoids this difficulty using a Monte-Carlo simulation (this is why it is often refered to as a "frequentist approach").

This paper is organized as follows : in the next section, we describe our method and compute all the statistical properties of our choice of angular power spectrum estimates. From section 3 and on we apply it to the forthcoming Auger observatory. In section 3, we present the relevant informations about this experiment that we need in the context of angular power spectrum estimation. In section 4, we discuss the constraints that the Auger Observatory can put on isotropic distribution of cosmic rays at ultra-high energy. At last, in section 5, we extend the analysis to the case of a large scale anisotropic distribution.

2   Angular power spectrum with a partial sky coverage

2.1   Generalities

The number of cosmic rays observed per unit solid angle $\mathrm{d}N/\mathrm{d}\Omega$ is a Poisson random variable in each direction $\vec{n}$, whereas considered as a function of $\vec{n}$, this

is a Poisson random process. We model it with the two dimensional quantity :

$$\frac{\mathrm{d}N}{\mathrm{d}\Omega}(\vec{n}) = \mathcal{N}(\vec{n}) = \sum_{i=1}^{N} \delta(\vec{n}, \vec{n}_i)$$

where δ is the Dirac delta function on the surface of the unit sphere, and $\vec{n}_i$ the position of the ith cosmic ray. The total number of cosmic rays observed is then $\int \mathcal{N}(\vec{n})\mathrm{d}\vec{n} = N$. This distribution follows a Poisson law $\mathcal{P}(\nu(\vec{n}))$ with an averaged intensity density in the direction $\vec{n}$:

$$\nu(\vec{n}) = \frac{N}{4\pi f_1} W(\vec{n})\left(1 + \Delta(\vec{n})\right)$$

where W is the relative coverage of the experiment varying from 0 to 1, $f_1 = \frac{1}{4\pi}\int W(\vec{n})\mathrm{d}\vec{n}$ the fraction of the sky effectively covered by the experiment, and Δ some continuous stochastic field that measures the departure from isotropy. The stochastic field Δ is assumed to have a zero expectation value :

$$\langle \Delta(\vec{n}) \rangle_r = 0$$

where we have introduced the average over all the possible realizations of the random phases of the Δ field. The expansion of Δ on the spherical harmonics basis is given by :

$$\Delta(\vec{n}) = \sum_{\ell \geq 0} \sum_{m=-\ell}^{m=\ell} a_{\ell m} Y_{\ell m}(\vec{n})$$

It is natural to introduce the Δ random field because of the probable stochastic nature of the cosmic ray sources distribution and propagation through magnetic fields. The possible anisotropies we want to characterize are *a priori* of random nature. In particular, many theoretical models of UHECRs anisotropies are based on at least partly random configurations of sources and magnetic fields. We are driven to interpret a particular set of events as one specific realization of the random process; and we are led to characterize the underlying random process properties through the angular power spectrum of the data of the only set of events we have.

Turning now to the two point correlation function of the Δ field, we assume this function to be only dependent on the angular distance between two points on the sphere :

$$\langle \Delta(\vec{n})\Delta^\star(\vec{n}') \rangle_r \equiv \xi(\vec{n} \cdot \vec{n}')$$

This is a very strong and important hypothesis as it is the basis of any power spectrum estimation, in the classical sense we want to give to it. In particular, this assumes that the statistical properties of Δ are the same over the whole celestial sphere. In the following, we will refer to this hypothesis as *spatial stationarity* by analogy with a stationary time dependent problem, where the notion of stationarity is used when the two point correlation function depends only on the time difference[2]. Making a simple expansion of $\xi(\vec{n} \cdot \vec{n}')$ onto the Legendre polynomials[3] :

$$\xi(\vec{n} \cdot \vec{n}') = \sum_{\ell} \frac{2\ell + 1}{4\pi} C_\ell P_\ell(\vec{n} \cdot \vec{n}') = \sum_{\ell,m} C_\ell Y_{\ell m}(\vec{n}) Y_{\ell m}^\star(\vec{n}')$$

and of $\langle \Delta(\vec{n}) \Delta^\star(\vec{n}') \rangle_r$ onto the spherical harmonics :

$$\langle \Delta(\vec{n}) \Delta^\star(\vec{n}') \rangle_r = \sum_{\ell_1,m_1} \sum_{\ell_2,m_2} \left\langle a_{\ell_1 m_1} a_{\ell_2 m_2}^\star \right\rangle_r Y_{\ell_1 m_1}(\vec{n}) Y_{\ell_2 m_2}^\star(\vec{n}')$$

we clearly see by identification that spatial stationarity leads to a diagonal covariance matrix of the $a_{\ell m}$ coefficients :

$$\left\langle a_{\ell_1 m_1} a_{\ell_2 m_2}^\star \right\rangle_r = C_{\ell_1} \delta_{\ell_1 \ell_2} \delta_{m_1 m_2}$$

where C_ℓ is the angular power spectrum of the fluctuations.

The angular power spectrum is therefore a two point correlation function in ℓ space. It gives information on the correlation between two angular directions separated by an angular scale $\simeq 1/\ell$ (in radians). For a Gaussian field Δ, the C_ℓ power spectrum characterizes completely the fluctuations of the field as the even order moments are obtained from the second order moment and the odd order moments are zero (Wick's theorem). In the general case, this is no longer true and higher order moments (three points correlation function and so on ...) are necessary in order to fully characterize the field. One should note however, that, if the observed comic ray sky is the result of a sequence of many random processes and assuming no single process dominates (e.g. the sky is not the result of a single dominant source with no magnetic field), the fluctuations will be of Gaussian nature according to the central limit theorem.

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[2] We use here the vocabulary of "spatial stationarity" rather than the one of "homogeneity" by arbitrary choice.

[3] We use here the spherical harmonics addition theorem :

$$\sum_{m=-\ell}^{\ell} Y_{\ell m}(\vec{n}) Y_{\ell m}^\star(\vec{n}') = \frac{2\ell + 1}{4\pi} P_\ell(\vec{n} \cdot \vec{n}')$$

In any case, higher order moments can be computed on the data and departure from a Gaussian behavior can be measured.

As outlined before, the data set we are dealing with is a *Poisson sample* of the random field Δ. Consequently, we have to introduce a second kind of average : the average over all possible sample configurations $\langle \cdot \rangle_P$. Therefore, from now on, we use the notation $\langle \cdot \rangle \equiv \langle \langle \cdot \rangle_P \rangle_r$ to express this double average over the possible configurations of N events and over the possible realizations of the Δ random field. From the elementary Poisson statistic properties, it is easy to show that :

$$\langle \mathcal{N}(\vec{n}) \rangle = \langle \langle \mathcal{N}(\vec{n}) \rangle_P \rangle_r = \langle \nu(\vec{n}) \rangle_r = \frac{N}{4\pi f_1} W(\vec{n})$$

and :

$$\langle \mathcal{N}(\vec{n}) \mathcal{N}(\vec{n}') \rangle = \left(\frac{N}{4\pi f_1} \right)^2 W(\vec{n}) W(\vec{n'})(1 + \langle \Delta(\vec{n}) \Delta(\vec{n}') \rangle_r)$$
$$+ \frac{N}{4\pi f_1} W(\vec{n}) \delta(\vec{n}, \vec{n}')$$

2.2 Definition of the spherical harmonic coefficients estimate

We want to build an estimate of the harmonic expansion coefficients of Δ. In the cases we are interested in, the field Δ is not measured uniformly over the whole celestial sphere. This is due to the non uniform exposure of cosmic ray experiments. For a single experiment, the knowledge is even limited to a given region in the sky and no information on Δ is available elsewhere. Moreover, in this given region, the exposure is not uniform and generally depends on declination. When combining data from two observatories, the exposure becomes full sky but non uniform. This is shown for instance with Sugar and AGASA coverage in (Anchordoqui et al. 2003), or with Auger Southern and Northern sites in (Sommers 2000).

All these configurations can be described through the introduction of the window field $W(\vec{n})$ that measures the relative exposure in the direction $\vec{n}$ on the sky. This field can even vanish in some regions. Thus, $\tilde{\Delta}(\vec{n}) = \Delta(\vec{n}) \times W(\vec{n})$ is the quantity we have access to experimentally and not simply $\Delta(\vec{n})$ as in the case of a uniform and full sky coverage. This has an immediate effect in the C_ℓ determination as we cannot compute the expansion of the field we intended to. We only have access to what is called the pseudo-power spectrum $\tilde{C}_\ell$ of the product of the two fields. A simple way to go back to the true C_ℓ from the measurement of $\tilde{C}_\ell$ was proposed for Cosmic Microwave Background analysis

by (Hivon et al. 2002) and has been widely used in this community for various experiments (Netterfield et al. 2002; Benoit et al. 2002; Hinshaw et al. 2003). We will soon show that the convolution kernel which mixes the modes of the angular spectrum we want to measure is the same as the one found in the framework of the CMB.

We denote our estimates $\tilde{a}_{\ell m}$ and we define them as :

$$\tilde{a}_{\ell m} = \int\limits_{4\pi} \mathrm{d}\vec{n}\, Y_{\ell m}^{\star}(\vec{n}) \frac{\mathcal{N}(\vec{n}) - \frac{N}{4\pi f_1} W(\vec{n})}{\frac{N}{4\pi f_1}}$$

Clearly, $\langle \tilde{a}_{\ell m} \rangle = 0$, as well as for the expectation value of the true coefficients.

2.3 The bias on the angular power spectrum estimate

For reasons that will soon become clear, we introduce the following coupling kernel as in (Hivon et al. 2002) :

$$K_{\ell m \ell' m'} = \sum_{\ell_1 m_1} w_{\ell_1 m_1} \int\limits_{4\pi} \mathrm{d}\vec{n}\, Y_{\ell' m'}(\vec{n}) Y_{\ell m}^{\star}(\vec{n}) Y_{\ell_1 m_1}(\vec{n})$$

where we have expanded the window field on the spherical harmonics basis. Let us also introduce the power spectrum of the window field :

$$\mathcal{W}_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |w_{\ell m}|^2$$

Turning now to the correlation between two multi-pole estimates, it is easy to show that :

$$< \tilde{a}_{\ell m} \tilde{a}_{\ell' m'}^{\star} > = \sum_{\ell_1 m_1} C_{\ell_1} K_{\ell m \ell_1 m_1} K_{\ell' m' \ell_1 m_1}^{\star} + \frac{4\pi f_1}{N} K_{\ell m \ell' m'}$$

We then estimate the power spectrum $\tilde{C}_\ell$ simply by taking the empiric average over m :

$$\tilde{C}_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{m=\ell} |\tilde{a}_{\ell m}|^2$$

This yields to :

$$\left\langle \tilde{C}_\ell \right\rangle = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \left(\sum_{\ell_1 m_1} C_{\ell_1} \left| K_{\ell m \ell_1 m_1} \right|^2 + \frac{4\pi f_1}{N} K_{\ell m \ell m} \right)$$

In (Hivon et al. 2002), it has been shown that the first term is equivalent to a mode-mode coupling matrix $M_{\ell \ell_1}$:

$$\left\langle \tilde{C}_\ell \right\rangle = \sum_{\ell_1} M_{\ell \ell_1} C_{\ell_1} + \frac{4\pi f_1}{N} \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} K_{\ell m \ell m}$$

where the $M_{\ell \ell_1}$ matrix elements are :

$$M_{\ell \ell_1} = \frac{2\ell_1+1}{4\pi} \sum_{\ell_2} (2\ell_2 + 1) \, \mathcal{W}_{\ell_2} \begin{pmatrix} \ell & \ell_1 & \ell_2 \\ 0 & 0 & 0 \end{pmatrix}^2$$

which makes use of the Wigner 3-j symbols. By expanding the second term onto the Wigner 3-j symbols, and after some manipulations, it is easy to show that :

$$\frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} K_{\ell m \ell m} = \frac{w_{00}}{\sqrt{4\pi}} = f_1$$

leading to :

$$\left\langle \tilde{C}_\ell \right\rangle = \sum_{\ell_1} M_{\ell \ell_1} C_{\ell_1} + \frac{4\pi f_1^2}{N}$$

We therefore have a simple and analytical link between our estimate and the true C_ℓ for a sky observed with a varying and/or incomplete exposure. Apart from a bias, our estimate is just the convolution of the true power spectrum by a kernel whose properties can be determined analytically from the shape of the window.

At last, in (Hivon et al. 2002), it is shown that the effect of M on a constant is a multiplication by the second moment of the window $f_2 = \frac{1}{4\pi} \int W^2(\vec{n}) \mathrm{d}\vec{n} = \sum_\ell \frac{2\ell+1}{4\pi} \mathcal{W}_\ell$. Therefore, we can go back to the angular power spectrum of the Δ field through :

$$\left\langle C_\ell^{\mathrm{exp}} \right\rangle = \sum_{\ell'} M_{\ell \ell'}^{-1} \left\langle \tilde{C}_{\ell'} \right\rangle = C_\ell + \frac{4\pi}{N} \frac{f_1^2}{f_2}$$

We see that the experimental power spectrum is unmixed and asymptotically unbiased. The bias term can be easily computed analytically and is purely induced by the finite number of arrival directions that are available, that is, purely induced by the Poisson statistics of $\mathcal{N}$.

2.4 The variance of the angular power spectrum estimate

From the fourth moment of $\mathcal{N}$ and the Wick's theorem, there is no difficulty to compute the correlation between four multi-poles estimates. However, this calculation is rather long and tedious, so we don't reproduce it in details. As in cosmic ray physics, the null hypothesis we want to test is isotropy, we are interested in $C_\ell = 0$. In this case, the result for the covariance on $\tilde{C}_\ell$ is found to be :

$$\text{Cov}(\tilde{C}_\ell, \tilde{C}_{\ell'}) = \left(\frac{4\pi f_1}{N}\right)^2 \frac{2\pi}{2\ell'+1} M_{\ell\ell'}$$

Therefore, the variance on the experimental power spectrum simply reads :

$$V(C_\ell^{\text{exp}}) = \sum_{\ell_1, \ell_2} M_{\ell\ell_1}^{-1} \text{Cov}(\tilde{C}_{\ell_1}, \tilde{C}_{\ell_2})(M_{\ell_2\ell}^{-1})^T = \left(\frac{4\pi f_1}{N}\right)^2 \frac{2\pi}{2\ell+1} M_{\ell\ell}^{-1}$$

2.5 Discussion

One might ask the question of the pertinence of measuring an angular power spectrum on a partial region of the celestial sphere and ask what is the link between such a *local* power spectrum and the *global* one. In fact, all of this discussion is linked to spatial stationarity of the random field. If the field is spatially stationary, then, a partial part of the sky if a fair sample and can allow to recover all modes provided of course that the matrix $M_{\ell\ell'}$ can be inverted, which is the case when the portion of the sky covered is larger than $\simeq 40\%$ as shown on Fig. 1. If the sky is not spatially stationary (which can be seen by comparing power spectra in various partial regions of the sphere), then our procedure cannot be applied and the recovered power spectrum is of course not valid for the whole sphere. In that case anyway, the non stationarity of the sky makes the notion of a single power spectrum on the sphere totally irrelevant and our procedure allows one to construct power spectra for different regions provided the fact that they are spatially stationary enough. We therefore see that in any case, computing a *local* version of the angular power spectrum is interesting as it provides a *global* information when it is meaningful (spatially stationary sky) or a *local* one if relevant.
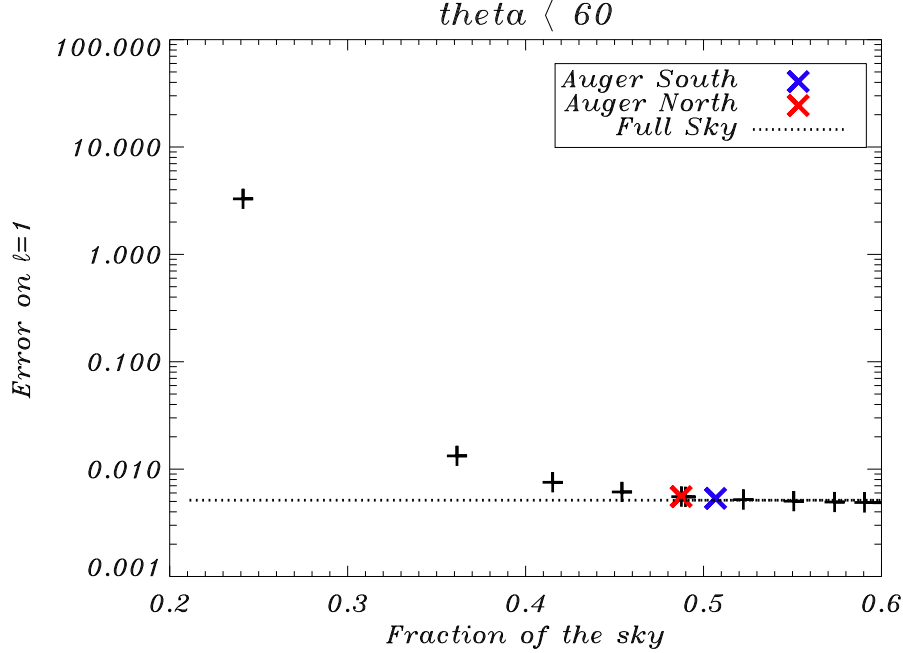
Fig. 1. Error bars of the reconstructed dipole as a function of covered portion of the sky. For an experiment covering more than 40% of the sky, the error bars become stable and the corresponding dipole estimate makes sense. For values smaller than 40%, the $M_{\ell_1 \ell_2}$ matrix is no longer regular and, as a result, the error bars explode. This conclusion arises with the conservative choice of θ_{max}=60 deg. See section 3 for full explanations about the parameter θ_{max}. Blue and red crosses show the error-bars for Auger South and Auger North respectively; the dotted the expected level for a full uniform sky coverage.

There are a lot of theoretical motivations to detect large scale patterns around 1 EeV as well as at higher energies. Magnetic fields or relative motion of the observer with respect to cosmic ray rest frame are natural mechanisms that lead to low-order moments. As these low-order poles characterize properties over the whole sphere, the condition of spatial stationarity should be naturally achieved in those cases, and local power spectra should show the same patterns whatever the observed portion of the sky. On the other hand, at higher poles, and if sources are located in large scale structure such as the Virgo cluster or the Super Galactic Plane, we expect that power spectra obtained from different regions of the sky covering or not those regions will show different characteristics. Again, spatial stationarity would be achieved within each region or along superstructures and the power spectrum obtained in those cases will give us informations on the sources distribution and magnetic fields within those structures. Of course, there is no way to completly describe the sky without looking at it. If a spectacular source or set of sources is present somewhere, the only way to know about it is to achieve (with several observatories) full sky coverage.

The formalism of the angular power estimation is up to now the tool the most sensitive to search for tiny anisotropies over the sphere, provided the fact that the detector doesn't smear out the arrival directions more than the scale $1/\ell$. One can imagine that the use of this observable in the next future will be of great help in order to bring strong constraints on the UHECRs models of production and propagation.

3   The Pierre Auger Observatory

The Pierre Auger Observatory [4] is the first of a new generation of detectors specially dedicated to the highest-energy cosmic rays. Large area ground based detectors do not observe the incident cosmic rays directly but the Extensive Air Showers (EAS), a very large cascade of particles, that they generate in the atmosphere. All experiments aim to measure, as accurately as possible, the direction of the primary cosmic ray, its energy and its nature. There are two major techniques used. One is to build a ground array of sensors spread over a large area, to sample the EAS particle densities on the ground. The other consists in studying the longitudinal development of the EAS by detecting the fluorescence light emitted by the nitrogen molecules which are excited by the EAS secondaries.

The Auger Observatory (The Pierre Auger collaboration 1995) combines both techniques. The detectors are designed to be fully efficient for showers above 10 EeV, with a duty-cycle of 100% for the ground array, and 10 to 15% for the fluorescence telescopes. The 1600 stations of the ground array are cylindrical Čerenkov tanks of 10 m^2 surface and 1.2 m height filled with filtered water; they are 1.5 km spaced on a triangular grid.

The Auger observatory covers both hemisphere, one site in the South is presently under construction in Argentina and will be completed at the end of 2005. The Northern site construction should start soon after. Once fully completed in 2008, the Auger Observatory will be covering a surface of 2×3000 km^2 and will provide unprecedented statistics. With a total aperture of more than 14000 km^2·sr, and with an integral cosmic ray intensity above 10 EeV of approximately 0.5/(km^2·sr·yr), the Auger Observatory should detect every year of the order of 7000 events above 10 EeV and 70 above 100 EeV (assuming a E^{-3} dependence of the spectrum). The angular resolution of the surface detector alone is believed to be of the order of 1 degree, and can be improved to less than 1 degree in the case of the so-called hybrid events, that

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[4] Named after the French physicist Pierre Auger (1899-1993) who discovered the Extensive Air Showers.

is, events detected by both the surface detector and the fluorescence detector (The Pierre Auger collaboration 1995).

A full-time operation of the surface detector means that there is no exposure variation in sideral time and therefore constant exposure in right ascension. For such a detector located at latitude a_0 and fully efficient for cosmic rays arriving with zenith angles θ less than some maximal value $\theta_{\max}$, the exposure is only a function of declination δ (Sommers 2000) :

$$W(\vec{n}) \propto \cos(a_0)\cos(\delta)\sin(\alpha_m) + \alpha_m \sin(a_0)\sin(\delta)$$

where α_m is given by :

$$\alpha_m = \begin{cases} 0 & \text{if } \xi > 1 \\ \pi & \text{if } \xi < -1 \\ \cos^{-1}(\xi) & \text{otherwise} \end{cases}$$

and :

$$\xi = \frac{\cos(\theta_{\max}) - \sin(a_0)\sin(\delta)}{\cos(a_0)\cos(\delta)}$$

Fig. 2 shows the resulting declination dependence for the two sites of the Auger Observatory located at latitude $a_0 = +39$ deg. for the Northern one, and $a_0 = -35$ deg. for the Southern one. The cut angle $\theta_{\max}$ is chosen at 60 deg. Also shown is the combined exposure, which will completely cover the celestial sphere but will not be uniform.

In order to show that analysis are not sensitive to the choice of $\theta_{\max}$ for the two latitudes we are considering, Fig. 3 plots, for a set of 7000 events, the variation of the first multi-pole $C_1^{\exp}$ error bars as function of $\theta_{\max}$. Clearly, for both sites from 50 deg. and on, the error bars are stable with respect to the $\theta_{\max}$ parameter.

4 Predicted constraints on isotropy with the Auger detector

In this section, we choose to deal with events with energy beyond 10 EeV, where the required fully efficient cosmic rays detection is satisfied by the Auger Observatory for a large range of $\theta_{\max}$. The total number of events $N_{\text{tot}}(E)$ being detected with the Auger arrays on the whole sphere is approximately
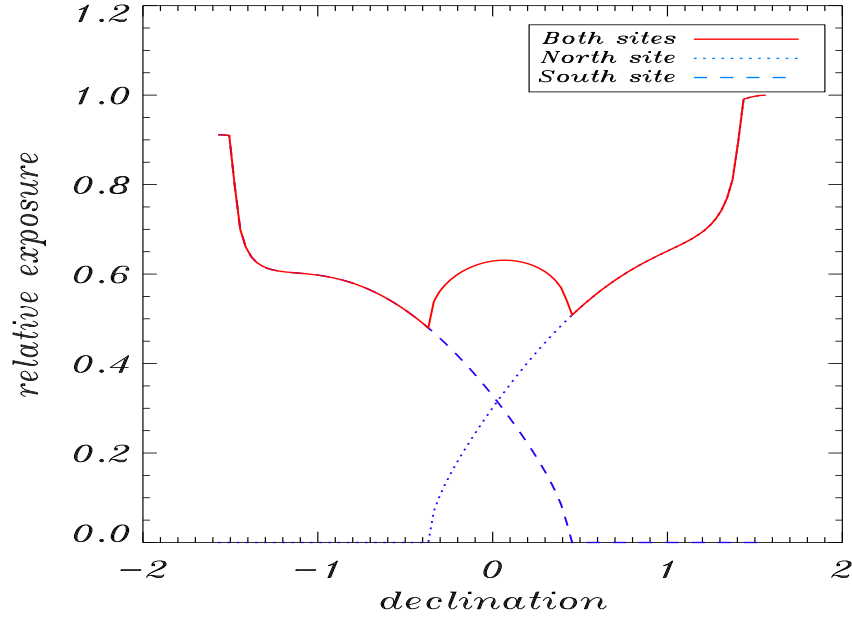
Fig. 2. The declination dependence of the Auger Observatory relative exposures. The Southern and Northern sites are indicated separately by dots, whereas the combined exposure is in solid line.
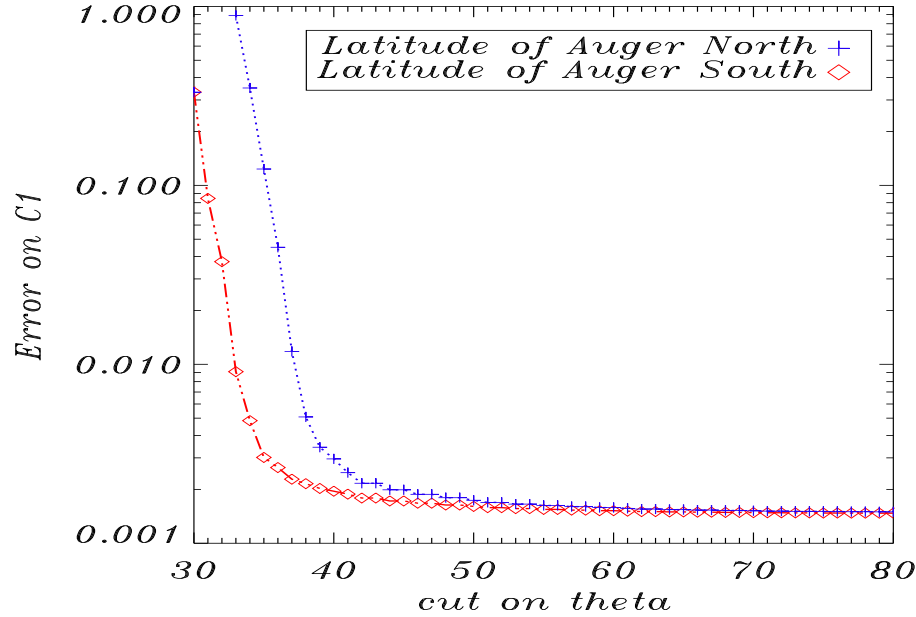

Fig. 3. Amplitude of the C_1^{exp} error bars as function of $\theta_{\max}$ for the two Auger sites latitudes, computed with a set of 7000 events. From $\theta_{\max} = 50$ degrees, results on the C_ℓ^{exp} error bars are not sensitive to the choice of $\theta_{\max}$.

$N_{\text{tot}}(E > 10\,\text{EeV}) = 7000$ per year. We consider here the number of events N_{ev} for an integration time T, weighted by the covered fraction of the sky :

$$N_{\text{ev}} = N_{\text{tot}}(E > 10\,\text{EeV}) \times T \times f_1$$

To check our estimate of the bias and of the error bars, we simulated 100 times N_{ev} events on a uniform (zero power spectrum) sky with the Southern Auger site coverage. We then reconstructed the power spectrum using the relations given in the previous sections. Fig. 4 shows the perfect agreement between the Monte-Carlo simulations and the analytical predictions for both the bias (the bias has been subtracted and the resulting values are indeed centered on zero) and the error bars. This plot is performed for an integration of 1 year of data taking.
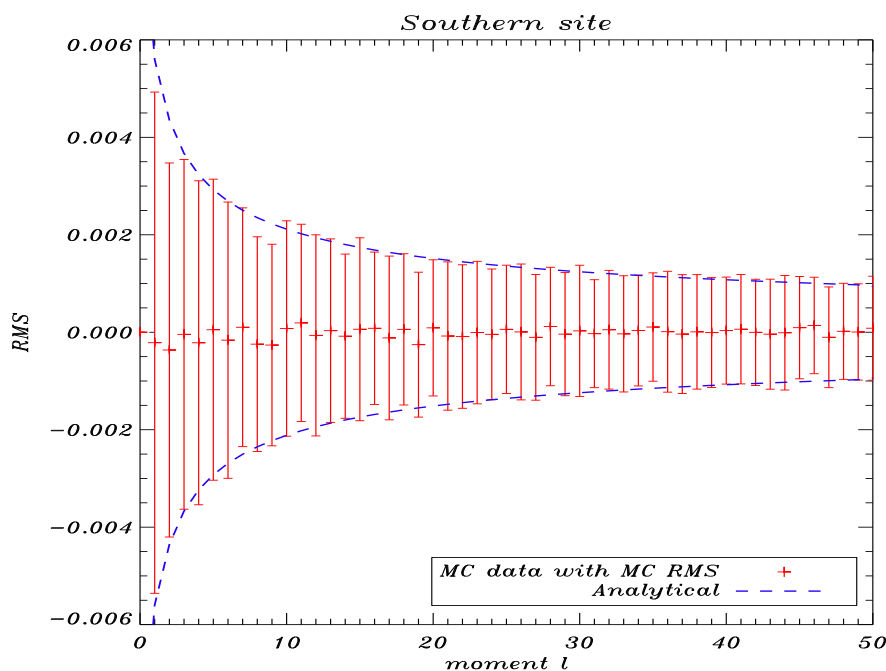

Fig. 4. Comparison between the analytical estimation of the bias and of the error-bars and the simulated ones of our angular power spectrum estimate. The coverage of the experiment is assumed to be the Auger Southern one, with a duration of 1 year data taking. We consider only statistics beyond 10 EeV. Clearly, the analytical computation perfectly reproduces the properties of the Monte-Carlo simulation.

The enhancement of the number of events with the Northern site allows for more stringent constraints on an isotropic distribution as can be seen on Fig. 5. We have imposed 3 years of data taking for the Northern site, and 6 years for the Southern one (3 years for the only Southern site + 3 years for both the Southern and Northern sites). With such statistics, it becomes possible to test the isotropy hypothesis with an accurate precision.
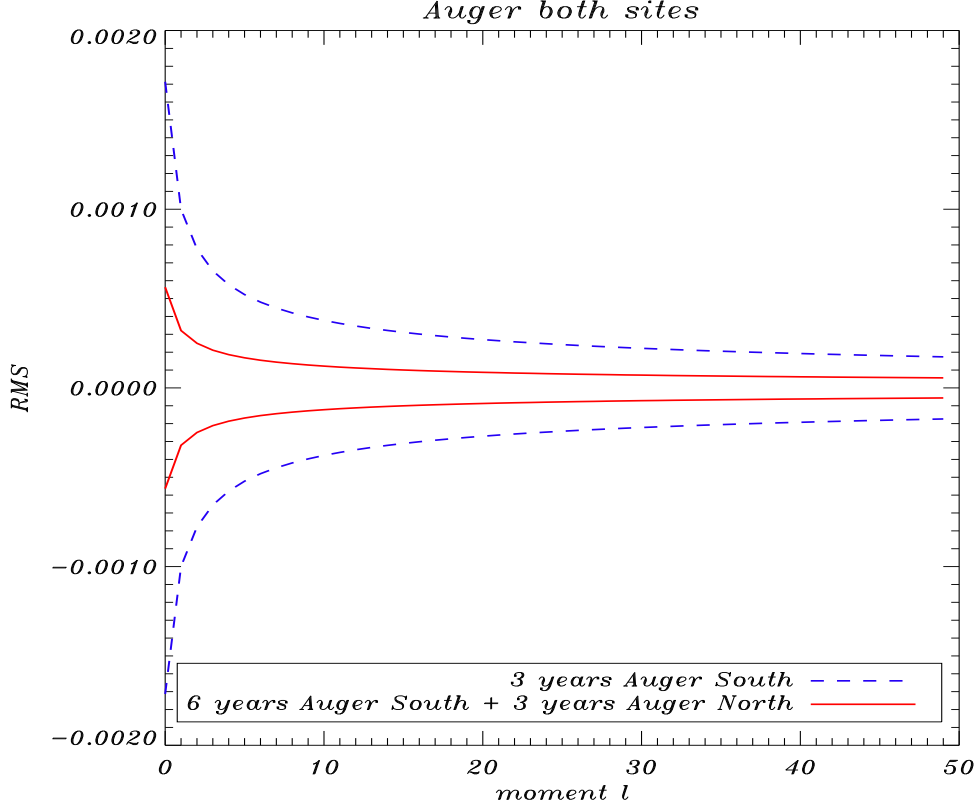
Fig. 5. Constraints on an isotropic distribution of cosmic rays beyond 10 EeV for 3 years of data taking of the Southern site only (dashed line), and for 6 years of data taking of the Southern site + 3 years of the Northern one (continuous line). These error-bars are estimated from the analytical computation.

5   Sensitivity of Auger to a dipole

In order to check for the efficiency of our mode deconvolution, we simulated events following a uniform distribution plus a dipole of 10 % amplitude. In practice, from the C_ℓ input spectrum, we generated the $a_{\ell m}$ coefficients with uniform random phases. We then transformed these coefficients into a sky map using an inverse harmonic transform in order to have a realization of the Δ random field. We then multiplied this map by the required coverage of the sky, and drew the number of events falling in each pixel using a Poisson law with average proportional to the this map. The exact position of each event within the pixel is drawn uniformly. All these steps rely heavily on the software provided along with the Healpix pixellisation scheme (Gorski et al. 1998). For each Monte-Carlo realization of a set of events for a given input power spectrum, we then extract the pseudo power spectrum by expanding the map onto the spherical harmonics basis (using the `anafast` routine) and then apply the deconvolution to reconstruct the unbiased and unmixed power spectrum. We did the simulations for the Southern site only and for both sites with the same

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integration of data taking than in the previous section.

The result of the Monte-Carlo are shown on Fig. 6. The reconstructed power spectrum is in agreement with the input one, and excludes the isotropic distribution at the 2.5 σ level using the Southern site only. Obviously, the improvement of the reconstructed power spectrum is clear with the 3 years data taking of the Northern site added to the analysis. As a consequence, the error-bars of the quadrupole are strongly reduced. The same procedure can be applied to any input power spectrum.
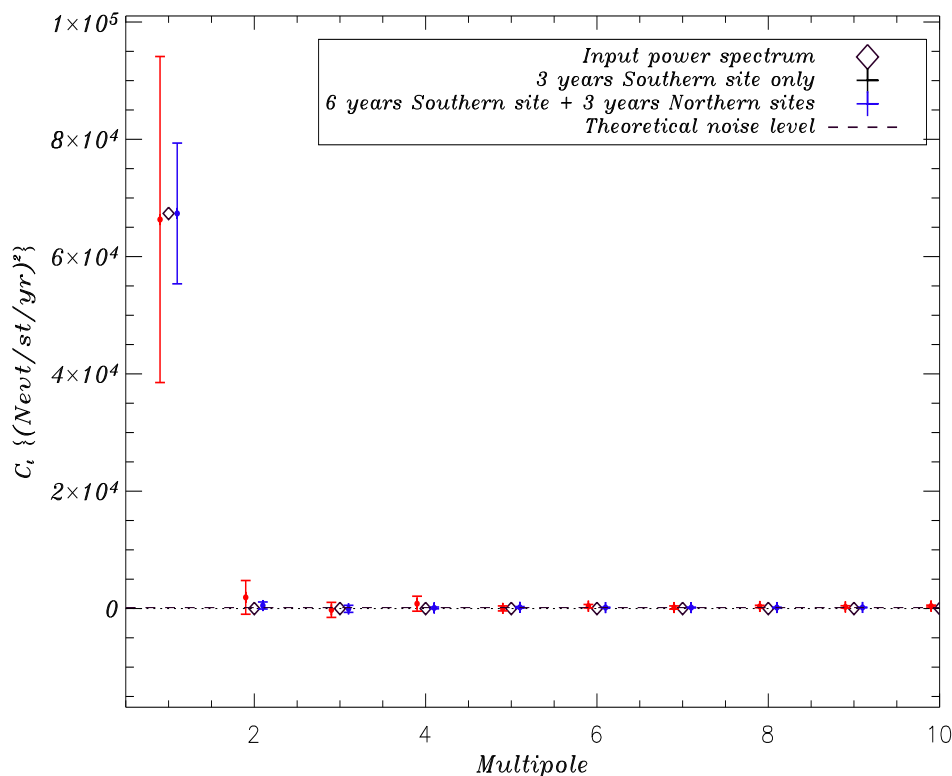

Fig. 6. Reconstructed power spectrum (expressed here in [Number of events/year/str]^2)in case of a pure dipole input sky. The diamonds show the input power spectrum. In red is shown the reconstructed power spectrum with the only Southern site of the Auger Observatory, for a duration of 3 years of data taking; whereas in blue is shown the reconstructed power spectrum with both sites, for a duration of 6 years for the Southern site and 3 years for the Northern one (as in previous section).

6 Conclusions

We showed that in the general case of a varying and incomplete exposure on the sky, the true power spectrum of the cosmic ray sources distribution can be

recovered. This result is not new in itself as it was introduced a few years ago in the framework of CMB data analysis (Hivon et al. 2002). Its application to cosmic ray data is however new and might open new possibilities as the general feeling up to now was that no C_ℓ power spectrum can be reconstructed without a complete sky coverage. The power spectrum that our procedure allows to recover is equivalent to the full sky one if the anisotropies in the arrival directions of the cosmic rays are well modeled by a spatially stationary random field on the sphere. If this is not the case, the recovered power spectrum is still valid, but only for the region that was used to determine it. Anyway in the non spatially stationary case, different power spectra are required in different regions of the sky and our approach is still relevant. Additionally, we have analytically solved the calculation of the bias and of the variance introduced by the finite sampling of the sky in the general case of a varying and eventually incomplete exposure. Using the deconvolution proposed here, any cosmic ray dataset will be usable for anisotropy determination purpose, provided the fact that the arrival directions and coverage map are known within reasonable precision.
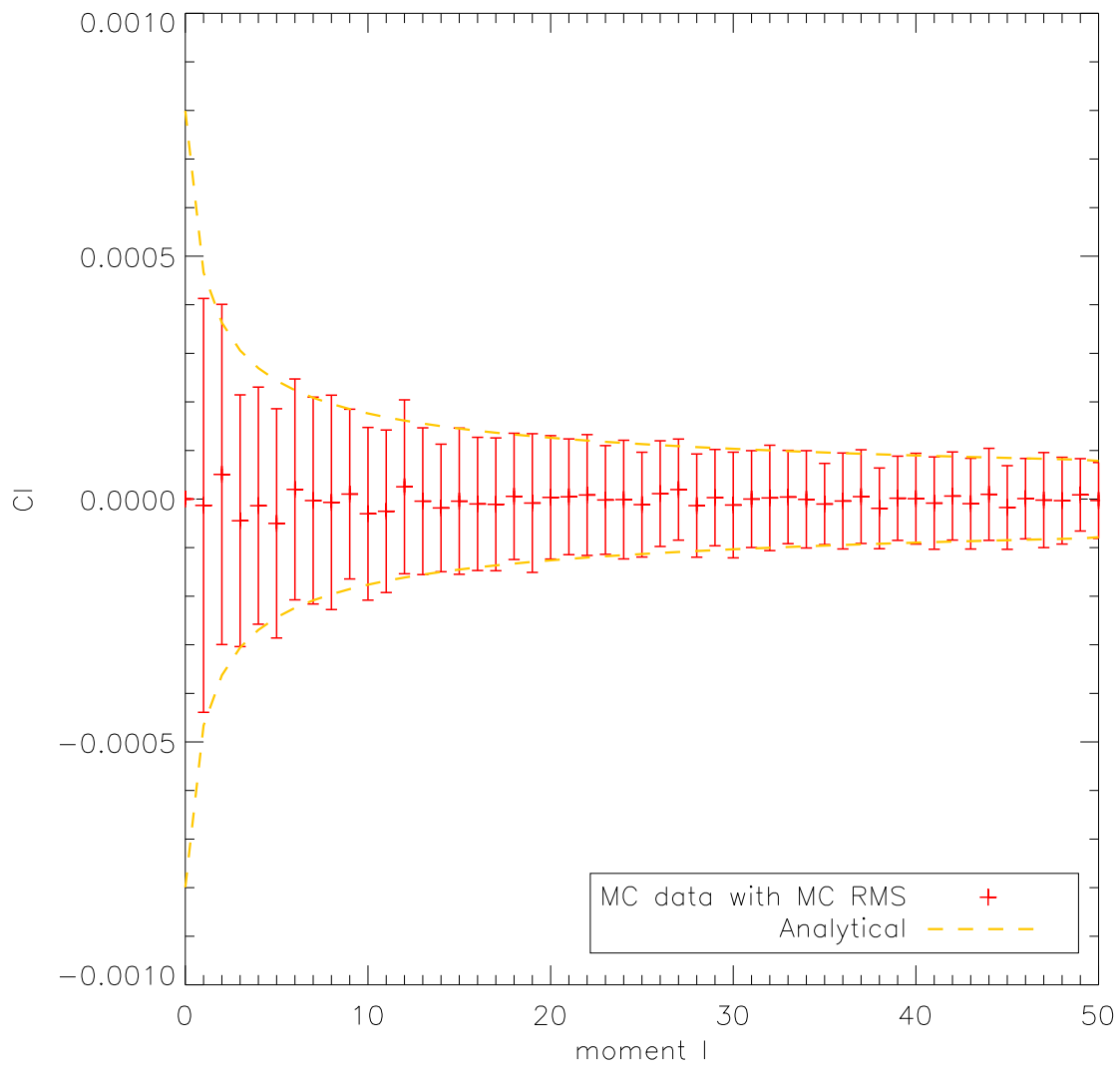
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Southern site